

# Hungarian mathematical culture: different interests, common features

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without antecedents

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  - Interests in the education of mathematicsProper method: discovery

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and two non-mathematicians who were influenced and have influenced Hungarian mathematical culture:

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- Competitions for students

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First decade of 20th century: Contributions to set theory  
(cardinality arithmetic)

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On LEM: it belongs to Gods logic, not to ours we dont have definite answers for every question

# Pólya and his heuristics



Georg Pólya (1887-1985)  
classical analysis, heuristics

# Pólya's 'return to philosophy'



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Mathematical intuition is not a mystic ability. It can be developed by the right way of mathematical education

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Logic (decision problem, consistency of arithmetics), computer theory



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Several philosophical writings, always connected with the problems of teaching mathematics, and always stressing on the first place the fallibility of mathematics

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- level of model construction (arithmetization)

This road must not be spared when teaching mathematics. We should teach - on university level - the exact, modern concepts (example: the  $\varepsilon - \delta$  definitions in analysis), but we must not forget about the intuitive basis of these concepts and about the reasons why they don't suffice.

# The consistency proof

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  - Kalmár: several month's work to display the why, the strategy, the costs and benefits of the proof to reach intuitive clarity above the logical exactness.
  - Published cca. 30 years later as 'Kalmár's proof' in the 2nd edition of Hilbert-Bernays, *Gundlagen der Mathematik* – although Kalmár insisted that it was just a reformulation of Gentzen's proof.

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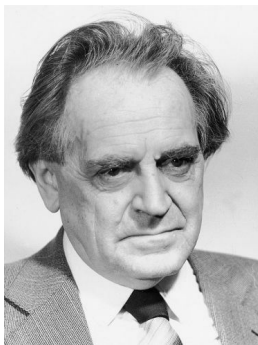
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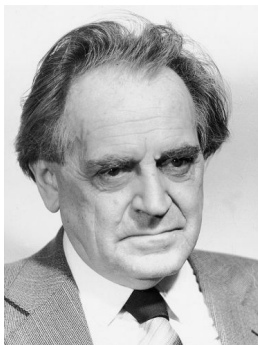
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"[M]athematical rigor can be acquired . . . only by developing the pupil's taste for rigor by starting with the intuitive point of view and showing repeatedly why some degree of rigor becomes necessary for certain problem."

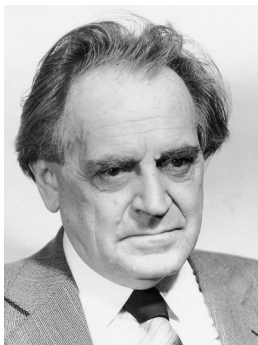


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Classic scholar, historian of mathematics



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After the '56 revolution: fellow of the academical institute for mathematical research (Rényi Institute)

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Dialectics and discovery method of teaching mathematics are intertwined.

# A many-sided talent



Alfréd Rényi (1921-1970)

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Probability theory, graph theory, number theory ...

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Probability theory, graph theory, number theory . . .

Philosophical writings in literary form: pseudo-Platonic dialogues, pseudo Pascal letters



# Rényi's fictionalism

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Analogy: persons of fiction:

'If we say that Clytemnestra was guilty, it means only that this is how Aeschylus imagined her and presented her in his play.

The situation is exactly the same in mathematics. We may be sure that the diagonals of the rectangle are equal because it follows from the definition of a rectangle given by mathematicians.'

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- A different philosophy of mathematics from Pólya, Kalmár and Lakatos - but fits into the 'humanist tradition' (R. Hersh).



# The dialectics of mathematics



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Second PhD: Cambridge, philosophy, supervisor: Pólya

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Kalmár: Teaching of mathematics should follow the historical development

Lakatos: History should follow the logic of education/discovery of mathematics

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Lakatos' criticism of 'formalism':

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Lakatos' criticism of 'formalism':

Mathematics is not just a set of theorems organized by the rules of deduction but a field of human activity.



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**Thank You for Your Attention!**